Total internal reflections in the analysis of downhole seismic testing data

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ABSTRACT: When performing seismic cone penetration testing near deep foundation elements or stone columns the incident angle of the source wave is likely to exceed the so-called critical angle. Whenever this occurs it is impossible to satisfy Snell's law (using real angles) and Total Internal Reflection (TIR) occurs, whereby the source wave is largely reflected from the boundary instead of being refracted. The analysis of downhole seismic testing data that contain TIRs is complicated by the fact that the presence of TIRs causes a seismogram to be recorded which contains the direct wave and several high amplitude and phase shifted reflections superimposed. In this paper the concept of TIRs and the associated source wave phase shift will be discussed and a proposed procedure will be given to allow the proper analysis of seismic data that contain TIRs

1 INTRODUCTION

Downhole Seismic Testing (DST) techniques such as the Seismic Cone Penetration Test (SCPT) are geotechnical *in-situ* site characterization tools, which enable direct shear (V_S) and compression wave (V_P) velocities to be accurately estimated. In DST the in-situ V_S and V_P velocities are determined by obtaining the corresponding source wave relative arrival times as the source wave travels within the stratigraphic profile. This is typically done by identifying reference points or markers within the seismograms (e.g., maximum peaks/troughs or crossovers) (Amini, 2006) or through techniques that implement the cross-correlation function or cross-power spectrum (Baziw, 1993, 2000; Baziw & Verbeek, 2013). For these approaches it is of paramount importance to have high quality first arriving source waves from each recorded trace so that meaningful relative arrival times are obtained.

Once the relative arrival times are derived the standard industry practice is then to assume straight ray travel paths between seismic source and receiver. In this case, the *in-situ* velocities are calculated as the ratio between the relative arrival time differences and the corresponding relative travel path differences between successive depths: $V = \Delta d/\Delta T$, where ΔT is the relative arrival time difference, Δd is the relative travel path difference and V is the corresponding interval velocity. Baziw (2002) and Baziw and Verbeek (2012) outline a more sophisticated DST approach in obtaining *in-situ* V_S and V_P velocities from relative arrival times. In this technique an Iterative Forward Modeling (IFM) method is utilized which takes into account source wave refraction as the source travels within the stratigraphic profile.

However, in DST investigations there are site conditions that result in wave multiples referred to as Total Internal Reflections (TIRs) that make the estimation of interval relative arrival times a very difficult task. TIRs are typically encountered whenever there are significant man-made structures nearby the test location (e.g. foundation piles, stone columns, or deep underground structures such as deep basements, parking garages, and dam structures).

This paper outlines the phenomenon of TIRs which are encountered within DST. In addition, a seismic signal processing technique is presented, which allows for processing seismograms which contains TIRs so that accurate relative and true interval arrival times can be obtained and subsequently interval velocities can be determined. This technique incorporates time variant Blind Seismic Deconvolution (BSDtv) where the direct source wave is separated from the overlapping/reflected waves. The relative and true interval velocities are then derived from the estimated direct source waves. The BSDtv filter formulation is referred to as BSDSolver-tv™.

2 BACKGROUND

2.1 *Total Internal Reflections*

The physics and governing mathematical equations of TIRs (Baziw & Verbeek, 2013; Aki & Richards, 2002) are outlined by considering a horizontally polarized (SH-wave) SCPT investigation within nearby concrete piles or stone columns. In this case the SH-wave velocity of the piles/stone columns (V_2) is much greater than that of the surrounding soil (V_1). The *precritical reflection coefficient* (reflections at angles less than the critical angle) is:

$$R = \frac{A_1}{A_0} = \frac{G_1\eta_1 - G_2\eta_2}{G_1\eta_1 + G_2\eta_2} = \frac{\rho_1 V_1 cos\theta_1 - \rho_2 V_2 cos\theta_2}{\rho_1 V_1 cos\theta_1 + \rho_2 V_2 cos\theta_2} \tag{1}$$

while the *post-critical reflection coefficient* can be given as

$$R = \frac{A_1}{A_0} = \frac{G_1\acute{\eta}_1 - iG_2\acute{\eta}_2}{G_1\acute{\eta}_1 + iG_2\acute{\eta}_2} = \frac{e^{-i\alpha}}{e^{+i\alpha}} = e^{-i2\alpha} \tag{2}$$

with

$$\alpha = tan^{-1}\left(\frac{G_2\acute{\eta}_2}{G_1\acute{\eta}_1}\right) \tag{3}$$

where R = the reflection coefficient, A_0 = the amplitude of incident, A_1 = the amplitude of reflected wave, ρ = the medium density, θ_1 = the incident and reflecting angles, θ_2 = the refraction angle, $G_1$and G_2 = the shear modulus of mediums 1 and 2, respectively, i = the imaginary number, and $\acute{\eta}_1$ and $\acute{\eta}_2$ = the *post-critical* vertical slowness within mediums 1 and 2, respectively, which can be given as:

$$\acute{\eta}_1 = \sqrt{u_1^2 - p^2} \tag{4a}$$

$$\acute{\eta}_2 = \sqrt{p^2 - u_2^2} \tag{4b}$$

where p = the ray parameter.

The seismogram for TIRs is generated by convolving the source waves by the reflection coefficients given in eq. (2) for various incident angles that exceed the critical angle. In the frequency domain this is equivalent to multiplying the source wave by the reflection coefficients given by eq. (2) producing a frequency-independent phase shift as shown below:

$$U = RA_0 e^{i(kx-\omega t)} = A_0 e^{-i2\alpha} e^{i(kx-\omega t)} = A_0 e^{i(kx-\omega(t+2\alpha/\omega))} \tag{5}$$

where k = the wave number, x = the distance, ω = the angular frequency and t = the time. As is evident from eq. (5), the reflected wave has a phase shift of $2\alpha/\omega$ or $2\alpha/2\pi f_d$ where f_d is the dominant frequency of the source wave.

A commonly utilized analytical source waves in seismic signal processing is the Berlage source wave. This wave is defined as (Baziw & Verbeek, 2013; Baziw, 2006, and 2011)

$$w(t) = AH(t)t^n e^{-\alpha t}\cos(2\pi f t + \emptyset) \tag{6}$$

where $H(t)$ = the Heaviside unit step function [$H(t) = 0$ for $t \leq 0$ and $H(t) = 1$ for $t > 0$]. The amplitude modulation component is controlled by two factors: the exponential decay term α and the time exponent n. These parameters are considered to be nonnegative real constants.

To provide illustrative examples of the phase shifting of seismic source waves due to TIRs eq. (5) was applied on a 55 Hz Berlage source wave. In Figures 1(a) and 1(b), the solid lines are the Berlage source wave, while the dotted lines the inputted Berlage source wave phase shifted by 70° and 230°, respectively. As is demonstrated in these figures, the phase shift does not impact the dominant frequency or the arrival time, but it does affect the minimum and maximum peaks, resulting in source wave distortion.

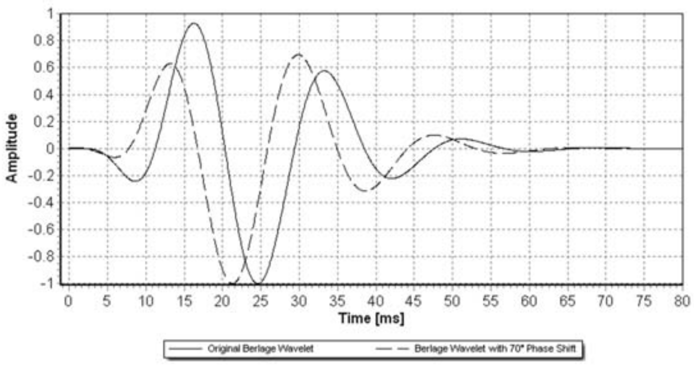

Figure 1(a): Berlage source wave - 70° phase shift.

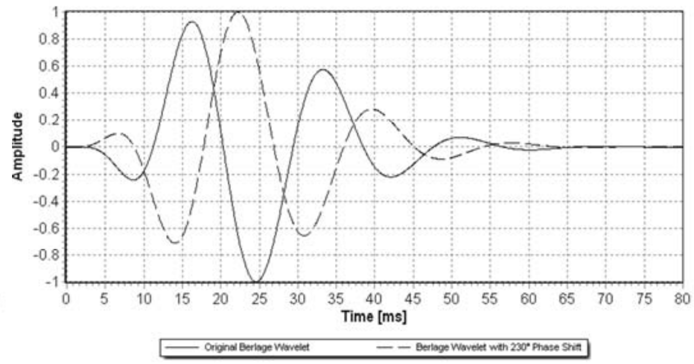

Figure 1(b): Berlage source wave - 230° phase shift.

DST seismograms that contain TIRs result in considerable challenges when obtaining relative or true arrival times for interval velocity calculations. The application of the standard relative arrival time techniques such as identifying reference points or markers within the seismograms or techniques that implement the cross-correlation function or cross-power spectrum are not feasible.

Figure 2 illustrates a vertical seismic profile (VSP) from simulated DST seismograms where stone columns are present nearby (Amini, 2006). The simulated seismograms are the responses of imaginary horizontal accelerometers, and the simulated trace at each depth is plotted along with its mirror image in order to mimic left and right SH-wave hammer beam impacts. The identified time markers (B to F) are not suitable as the basis for analyzing the seismograms due to the fact that their corresponding time location are dependent upon the constructive and destructive interference of the generated source waves, which in turn depend on the geometry of the higher velocity stone columns, location of the source, and depth of the seismic adapter. These parameters will be unique for each DST test and therefore a standardized analysis method cannot build on these markers.

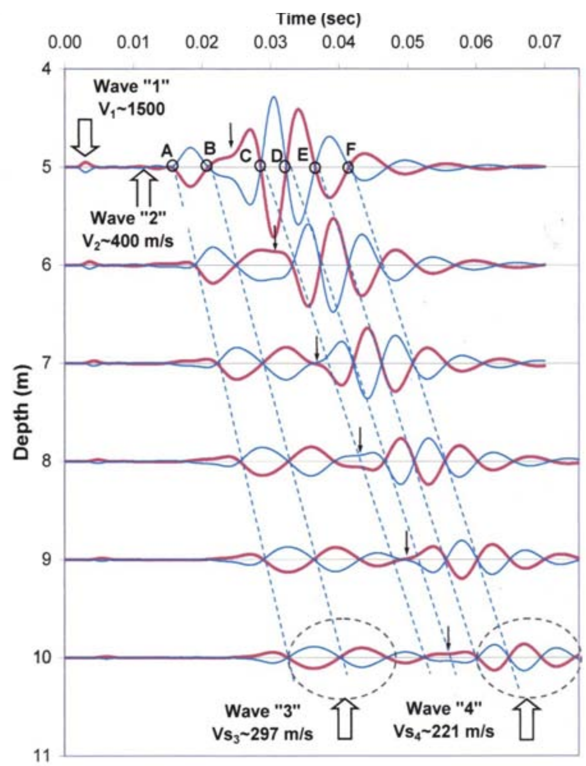

Figure 2. DST time markers for SCPT carried out in stratigraphy containing stone columns (Amini (2006)).

It should also be noted that the testing configuration implemented to generate Figure 2 relied upon exciting both the stone columns and the soil profile under analysis, which further increases the complexity of the generated source waves. It is therefore highly recommended that in DST only "point sources" are utilized (with beams of short length in case of SH-wave analysis). In doing so only the soil under analysis is excited and not the stone columns, and consequently only the direct SH-wave and corresponding reflection will be recorded.

2.2 *Time Variant Blind Seismic Deconvolution*

Seismic deconvolution is one of the most widely researched and implemented seismic signal processing tools (Ulrych & Sacchi, 2005; Gholami & Sacchi, 2012). In seismology, the recorded time series, *z(t)*, is defined as the linear convolution of the source wave, *S(t)*, with the earth's reflection coefficients, $\mu(t)$, with additive measurement noise, *v(t)*. The mathematical representation of this relationship is given as

$$z(t) = \int_0^t \mu(t)S(t-\tau)d\tau + v(t) \tag{7}$$

The discrete representation of (7) is given as

$$z(k) = \sum_{i=1}^{k} \mu(i)S(k-i) + v(k), \quad k = 1, 2 \cdots N \tag{8}$$

Blind seismic deconvolution (BSD) has to be used where both *S(t)*, and $\mu(t)$ are unknown, but whenever the source wave is not stationary (as is the case when TIRs are present) simple BSD is a not adequate. In that case a more advanced version of this technique has to be applied: Time Variant BSD (BSDtv). The BSDtv filter formulation outlined in this paper is referred to as BSDSolver-tv™.

3 BSDSOLVER-TV™ FILTER FORMULATION

3.1 *PPD algorithm*

Baziw (2006 and 2011) outlines a powerful BSD algorithm referred to as Principle Phase Decomposition (PPD™). In the PPD™ algorithm the source wave is modeled as an Amplitude Modulated Sinusoid (AMS): a sinusoid with a dominant frequency and phase modulated by an Amplitude Modulating Term (AMT). The discrete representation of the AMS source wave is given as

$$AMS_k = AMT_k \sin[2\pi f \Delta k + \varphi] \tag{9}$$

where k = the time index, f = the dominant frequency, Δ = the sampling rate, and φ = the phase of the AMS source wave.

A variant of the PPD™ algorithm which incorporates iterative forward modeling (PPD™ -IFM) has been demonstrated (Baziw, 2011) to provide very accurate estimations of the source wave and corresponding reflection coefficients when processing noisy seismograms with stationary reflected source waves (standard BSD). The PPD™-IFM algorithm is reformulated for the time variant BSD case (TIRs) by taking into account additional *a prior* information for TIRs. As in the case with all optimal estimation solutions, the greater the *a priori* knowledge specified directly results in more accurate estimates. In this case the following *a priori* information for TIRs is incorporated:

- although the phase of the source wave changes for each reflection, the dominant frequency remains the same;
- the time width of the source wave remains constant;
- the reflected source waves are modeled as AMSs;
- the AMTs of the direct and reflected source waves remain constant.

The BSDtv filter formulation which applies a variation of the PPD™ -IFM algorithm is referred to as BSDSolver-tv™. The BSDSolver-tv algorithm processes 2 to 3 seismic traces simultaneously utilizing dual-core and quad-core CPU technology. In case of DST this means that two to three adjacent depth traces within the vertical profile (e.g., traces recorded at depths 7, 8 and 9 meters) are processed simultaneously. This is due to the fact that there will be minimal variation between the recorded source waves recorded at adjacent depths.

The first arriving or direct source waves for the seismic traces under analysis are estimated where a cost function defined at the weighted Root Mean Square Difference (RMSD) between the measured seismograms and the synthesized seismograms is minimized. The weight of the cost function is defined as the RMSD between the source wave parameters of dominant frequency (f), t_{max} (the time location of the maximum value of the AMT), and t_{width} (the time duration of the estimated source wave). Relative or true arrival times are then estimated from the extract first arriving or direct source waves utilizing the standard DST techniques of identifying reference points or markers or implementation of the cross-correlation function.

3.2 *BSDSolver-tv™ Simulation Results*

The implementation and performance of the BSDSolver-tv™ algorithm is demonstrated by considering the analysis of three challenging DST synthetic seismograms. The seismograms were outlined by Baziw and Verbeek (2013) and represent a simulated SCPT containing TIRs. This data set is challenging due to the fact that the direct and reflected source waves are time variant and there are five closely spaced equivalent reflection coefficients with dipoles.

In this DST dataset Berlage source waves were generated and superimposed at depths of 5m, 6m, and 7m. All simulated waves had the common values of $f = 70$ Hz, $n = 2$, and $\alpha = 270$ specified. The phase values were set at $\emptyset = 20°, 40°, 140°, and\ 250°$, respectively. The arrival times and maximum amplitudes were set as follows for the five Berlage source waves:

- at a depth of 5 m: (5 ms, 1), (8 ms, 0.75), (11 ms, 0.625), (17 ms, 0.8), and (22 ms, 0.65), respectively (see Figure 3).
- at a depth of 6 m: (9 ms, 1), (14 ms, 0.75), (17 ms, 0.625), (21 ms, 0.8), and (24 ms, 0.65), respectively (see Figure 5).
- at a depth of 7 m: (14 ms, 1), (17 ms, 0.75), (21 ms, 0.625), (24 ms, 0.8), and (29 ms, 0.65) respectively (see Figure 7)

For each depth superposition of the source waves is shown with the reflection coefficients to allow for the visualization of the arrival time and maximum amplitude values (Fig. 4 –"Seismogram 1" (S1), Fig. 6 –"Seismogram 2" (S2), and Fig. 8 –"Seismogram 3" (S3), for the depth of 5 m, 6 m and 7 m respectively). For this VSP simulation a source-sensor radial offset of 1.5m was assumed; therefore the slant distances, d_i, for the simulation depth of 5 m, 6 m and 7 m are 5.22 m, 6.18 m and 7.16 m, respectively. Assuming a straight ray travel path (i.e., no refraction) the first arriving direct source wave interval velocities can be calculated as $V_{5-6} = (6.18 - 5.22)/(9 - 5) = 241\ m/s$ for depth interval 5m to 6m and $V_{6-7} = (7.16 - 6.18)/(14 - 9) = 196\ m/s$ for depth interval 6m to 7m.

Figure 9 illustrates the BSDSolver-tv™ estimated Berlage Wavelet 1 for seismograms S1, S2 and S3 and the corresponding averaged seismogram superimposed upon the true Berlage Wavelet 1. As is shown in Figure 9, there is very close agreement between the estimated, averaged and true Berlgae Wavelet 1. Figure 10 illustrates the VSP of the estimated Berlage Wavelets shown in Figure 9. The seismic trace displayed at 4m is the averaged Berlage Wavelet 1 illustrated in Figure. 9.

As mentioned previously, the Cross-Correlation Technique (*CCT*) is a standard methodology in obtaining DST interval velocities. The CCT is based upon cross-correlating the waves recorded at consecutive depth increments. The value of the time shift at the maximum cross-correlation value is assumed to be the relative travel time difference for the wave to travel the depth increment. Cross-correlating the estimated Berlage Wavelet 1 for seismograms S1, S2 and S3 results in the relative arrivals times of 3.95 ms (true value = 4ms) and 5.08 ms (true value = 5 ms) for depth increments 5 m to 6 m and 6 m to 7 m, respectively. The corresponding interval velocities assuming a straight ray travel path are calculated as

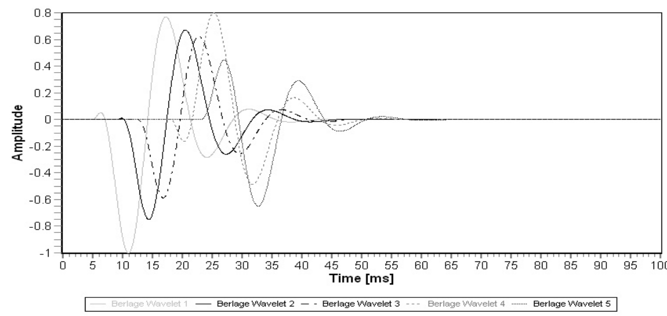

Figure 3: Simulation 1 - Berlage source waves with varying phases waves (Baziw & Verbeek, 2013).

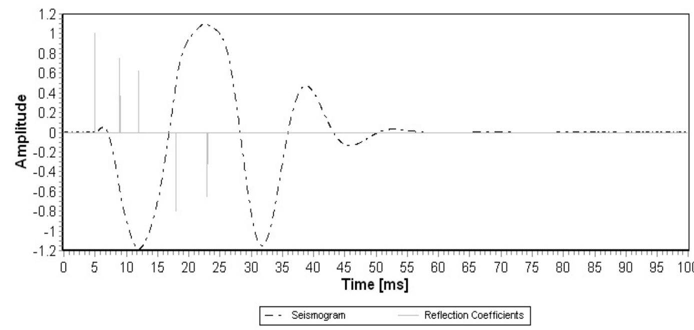

Figure 4: Superposition of Berlage source waves illustrated in Fig. 3 waves (Baziw & Verbeek, 2013).

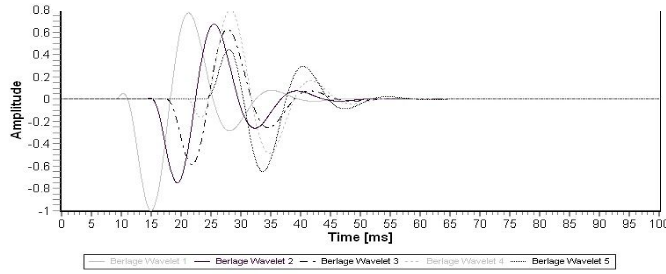

Figure 5: Simulation 2 - Berlage source waves with varying phases waves (Baziw & Verbeek, 2013).

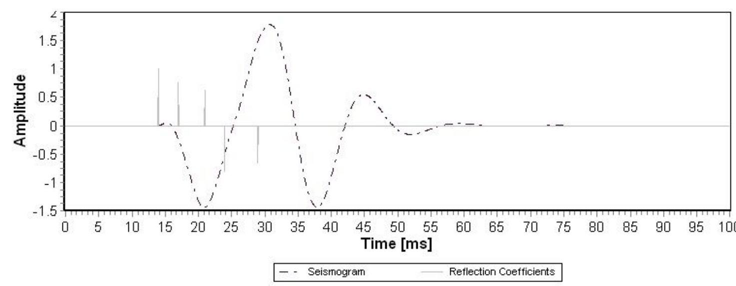

Figure 6: Superposition of Berlage source waves illustrated in Fig. 5 waves (Baziw &Verbeek, 2013).

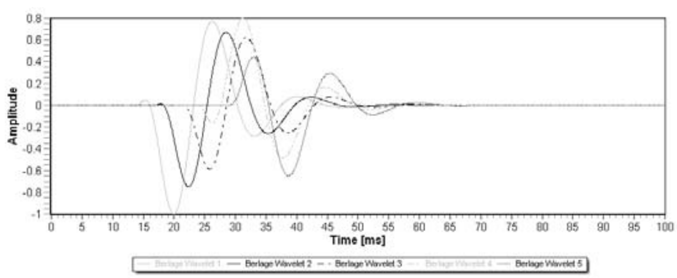

Figure 7: Simulation 3 - Berlage source waves with varying phases waves (Baziw & Verbeek, 2013).

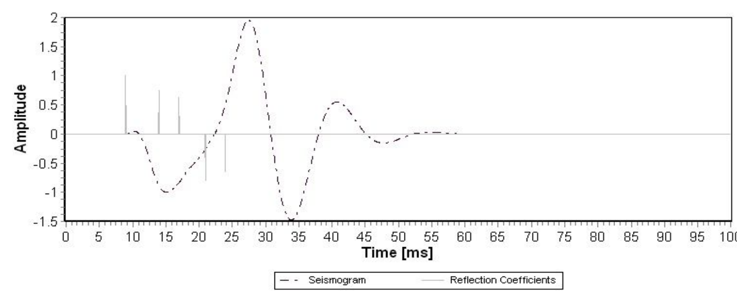

Figure 8: Superposition of Berlage source waves illustrated in Fig. 7 waves (Baziw & Verbeek, 2013).

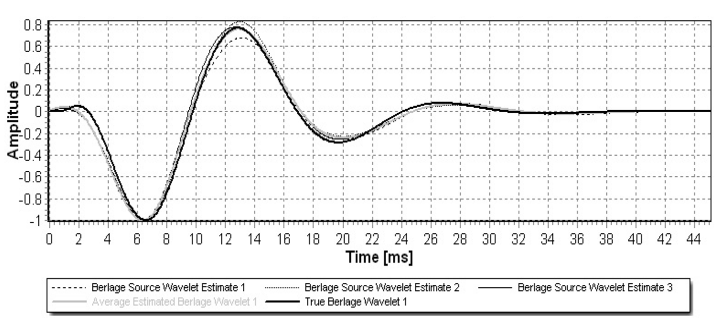

Figure 9: Estimated Berlage Wavelet 1 for seismograms S1, S2 and S3 and the corresponding averaged seismogram superimposed upon the true Berlage Wavelet 1.

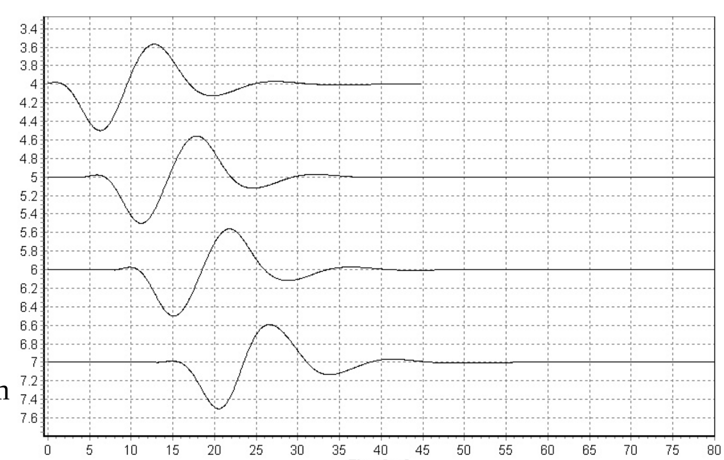

Figure 10. VSP of estimated Berlage Wavelets shown in Figure 9. The seismic trace displayed at 4m is the averaged Berlage Wavelet 1 illustrated in Figure 9.

$V_{5-6} = (6.18 - 5.22)/(3.95) = 243\ m/s$ for depth interval 5m to 6m and $V_{6-7} = (7.16 - 6.18)/(5.1) = 192\ m/s$ for depth interval 6 m to 7 m. This compares very close to the true values of $V_{5-6} = 241\ m/s$ and $V_{6-7} = 196\ m/s$

As outlined by Baziw & Verbeek (2013), it is not possible to accurately obtain relative arrival times utilizing time markers or the CCT when processing undeconvolved seismograms containing TIRs. For example, cross-correlating seismograms S1, S2 and S3 results in the relative arrivals times of 3.1 ms (true value = 4 ms) and 3.8 ms (true value = 5 ms) for depth increments 5 m to 6 m and 6 m to 7 m, respectively. The corresponding interval velocities assuming a straight ray travel path are calculated as $V_{5-6} = (6.18 - 5.22)/(3.1) = 310\ m/s$ for depth interval 5 m to 6 m and $V_{6-7} = (7.16 - 6.18)/(3.8) = 258\ m/s$ for depth interval 6 m to 7 m. These values for the interval velocities have corresponding percent errors of 29% and 32%, respectively.

4 CONCLUSION

As outlined in this paper, a very common and important problem encountered in DST is the recording of seismograms which contain Total Internal Reflections (TIRs). TIRs arise when the incident angle exceeds the critical angle, and they are associated with reflected source wave distortions due to the fact that the reflection coefficients become complex. In DST investigations TIRs are encountered whenever there are significant adjacent man-made structures (piles, stone columns, and deep underground structures such as deep basements, parking garages, and dam structures). Typical DST interval velocity estimation techniques which rely upon the implementation of the cross-correlation function, cross-power spectrum and/or time markers are unlikely to provide acceptable estimates when TIRs are present within the seismograms.

The processing of seismograms which contain TIRs requires time variant Blind Seismic Deconvolution (BSDtv). BSDtv refers to the case where there is an unknown direct source wave and time variant overlapping (reflected) source waves recorded by the seismogram. The goal of BSDtv is to separate the overlapping source waves. This paper outlined a BSDtv algorithm referred to as BSDSolver-tv™. And the implementation and performance of this algorithm was demonstrated by considering synthetic seismograms consisting of five closely spaced overlapping source waves with equivalent reflection coefficient dipoles, making them very challenging. It was shown that the BSDSolver-tv™ algorithm was able to obtain accurate estimates of the direct source wave and corresponding interval velocities for a VSP investigation.

5 REFERENCES

Aki, K., & Richards, P.G., 2002, "*Quantitative Seismology* (2nd Edition)", Sausalito, CA: University Science Books.

Amini, A., 2006, "*Application of Seismic Cone for Characterization for Ground Improved by Vibro-Replacements*" Ph.D. Thesis, Dept. of Civil Engineering, University of British Columbia.

Baziw, E., & Verbeek, G.E.H., 2013, "Signal Processing Challenges when Processing DST and CST Seismic Data containing TIRs ", submitted to the Geotechnical Testing Journal.

Baziw, E., & Verbeek, G.E.H., 2012, "Deriving Interval Velocities from Downhole Seismic Data", *Geotechnical and Geophysical Site Characterization 4* – Mayne (eds), CRC Press, pp. 1019–1024.

Baziw, E., 2011, "Incorporation of Iterative Forward Modeling into the Principle Phase Decomposition Algorithm for Accurate Source Wave and Reflection Series Estimation". *IEEE Transactions on Geosci. Remote Sensing* (TGRS), Vol. 49, No. 2, pp. 650-660.

Baziw, E. , 2002, "Derivation of Seismic Cone Interval Velocities Utilizing Forward Modeling and the Downhill Simplex Method", *Can. Geotech. Journal*., Vol. 39, pp.1-12.

Baziw, E. & Ulrych, T.J., 2006, "Principle Phase Decomposition - A New Concept in Blind Seismic Deconvolution", *IEEE Transactions on Geosci. Remote Sensing* (TGRS), Vol. 44, No. 8, pp. 2271-2281.

Gholami, A, & Sacchi, M.D., 2012, " A fast and automatic sparse deconvolution in the presence of outliers", *IEEE Transactions on Geosci. Remote Sensing* (TGRS), Vol. 50 , No. 10, pp. 4105 – 4116.

Ulrych, T.J., & Sacchi, M.D., 2005, "*Information-Based Inversion and Processing with Applications* (1st edition)", Amsterdam, The Netherlands: Elsevier B.V.